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EQUILIBRIUM CONDITIONS FOR THE AVERAGE STRESSES MEASURED BY X-RAYS

by

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EVANSTON, ILLINOIS

Equilibrium Conditions for the Average Stresses Measured by X-Rays

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I. C. Noyan*

ABSTRACT

True macro-stresses are measured with x-rays after material processing, only if there is a uniform plastic deformation in the sampled volume that is different than that in the rest of the material. In this stress-field the components in the direction of the surface normal are usually negligible over the depth penetrated by x-ray (for example, peening). However, pseudo-macrostresses, or backstresses, arise when this condition is violated, for example, if there are second phase particles and there is a gradient of plastic deformation from particle to matrix. This pseudo-macrostress field is three-dimensional and the stresses in the direction of the surface normal can be measured with x-rays. Equations are presented which allow estimates to be made of the magnitudes of the two kinds of stress fields.

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INTRODUCTION

Recently, several authors[1,2,3] have been able to detect the presence of stresses normal to the surface from x-ray measurements of peak-shifts. This has been possible because x-rays penetrate to a finite depth and the peak shifts depend on the strains averaged over these depths. Thus, stress components in direction "3" normal to the surface (which must be exactly zero at the surface) can contribute to peak shifts. The presence of such components is expressed in interplanar spacing $(d)_{hkl}$ vs $\sin^2\psi$, where ψ is the tilt of the specimen from the normal Bragg-Brentano focussing position on a diffractometer; this plot is curved when σ_{33} is significant and splits into two branches for $\pm\psi$ when σ_{13} and/or σ_{23} are important. Such stresses can arise during wear(4), grinding(3), shot-peening(2), carburizing, etc. and so this is a practically important topic.

Van Baal(5) and Brakman(6) have suggested that stress components normal to the surface are not required to produce ϵ_{13}, ϵ_{23}, ϵ_{33}, but rather that these strains (and their associated effects on d vs $\sin^2\psi$) arise due to elastic anisotropy in the presence of preferred orientation. Such anisotropy can produce these strains with only stresses parallel to the surface (σ_{11}, σ_{22}). In fact they conclude that normal stresses in direction "3", and, as well, σ_{12}, can not exist in ground samples. In reaching this conclusion, they invoke the differential equations of equilibrium at a point in a solid. However, x-ray measurements average over a [illegible] volume and these equations are not appropriate in this case. In fact, there are experiments that show this(2): When a sample, which was bent during grinding, was straightened, the shear stresses did not change, but there were large changes in the normal stresses. The coupling of strains and stresses through elastic constants, as in the approach by Van Baal and Brakman, require changes in all components of the

strain and stress tensors in such a situation. Furthermore, splitting (for $\pm \psi$) is observed for mild steel but not for Armco iron when the same grinding parameters are used in processing the samples[2].

We will show in this paper, that when there are "hard" and "soft" regions in a material, (such as matrix and precipitates, grain boundaries and grain interiors, regions of high and low dislocation density, etc.), the appropriate averaging of the equations for force equilibrium over the volume sampled by x-rays, leads to solutions which include stress components normal to the surface. We will use the approaches and results developed by Eshelby[7], Tanaka and Mori[8,9,10] and Mura[11] for this purpose.

THEORY

This will be presented in two parts: In the first part some assumptions inherent to the x-ray measurement of residual stress and the equations of equilibrium applicable to x-ray stress measurement will be discussed. In the second part, the total stress state formed in the near-surface layers of materials containing hard and soft regions (which arises when the surface layers are plastically deformed) is discussed.

A. The X-ray Measurement of Residual Stress

Despite the title of this section, we remind the reader that x-ray methods measure strains in the surface layers of a material. These strains are then converted into stresses using various assumptions as to the stress state and anisotropy (preferred orientation) existing in the surface layers[12,13].

The basic principle of obtaining the strains is simple[12]; the interplanar spacings of a specific form of planes are obtained from grains at different orientations to the surface normal. This is achieved by tilting and rotating the specimen with respect to the incident beam (Fig. 1). These spacings are

then converted into strains with the formula:

$$\varepsilon_{\phi\psi} = \frac{d_{\phi\psi} - d_o}{d_o} \qquad (1)$$

Where $d_{\phi\psi}$ is the spacing in the direction, $L_{\phi\psi}$, defined by the angles ϕ, ψ (figure 2) and "d"$_o$ is the spacing of the stress-free material.

Two major assumptions are included in the above treatment:

1) Since the strains from different grains are used to calculate the stress tensor[12,13], the stresses are assumed to be constant, or varying indentically, in all the grains sampled.

2) The spacings $d_{\phi,\psi}$ in equation (1) are obtained from the angular value of the Bragg peak corresponding to the diffraction planes. This value is an average value, over the total penetration volume of the x-rays. Only strain fields that are constant, or slowly varying, in the measurement volume contribute to the position of the Bragg peak. Rapid fluctuations cause broadening of the peak but do not affect the peak position[13,14]. Thus the stresses determined from the x-ray data are average values and may not represent the stress at a specific point in the volume of the measurement.

3) In measurements on multi-phase materials there is another limitation: If the structure and/or the lattice parameter of phases existing in the measurement volume are different, the average strains of only one phase is obtained with the use of a given reflection. The strains in the other phases must be determined by the use of their characteristic reflections. This property permits the determination of piecewise-continuous strain distributions; i.e. when the strains are continuous in respective phases but not across phase boundaries (figure 3).

Now, all internal stresses existing in a free body at equilibrium, with no surface tractions applied at the surfaces must obey the following differential equations of equilibrium:

$$\sigma_{ij,j} = 0 \quad , \qquad (2\text{-}a)$$

In the body volume, and,

$$\sigma_{ij} \cdot n_j = 0 \quad , \qquad (2\text{-}b)$$

at the surface of the body. Here n_j is the unit normal vector at the boundary (surface) of the body and summation over repeated indices is implied. Eqn. (2a) is the force balance of an infinitesimal element in the body and (2b) is the force balance if the infinitesimal element has a surface boundary. If no surface tractions exists, (2b) must be zero for the body to be at equilibrium. (Van Baal and Brakman have used eqn. (2a) to conclude that stresses in the direction of the surface normal can not exist for ground specimens.)

From eqn. 2 it can be shown that the average of any stress over the whole body is zero[11]:

$$\int_D \sigma_{ij} \, dD = 0 \quad , \qquad (3)$$

Where D is the total volume of the body. It must be emphasized that equation (3) is valid if and only if eqns. (2a) and (2b) are satisfied. This is easily shown[11]:

Integrating (3) by parts, and with "X" the distance coordinate in the "j" direction:

$$\int_D \sigma_{ij} \, dD = \int_{|D|} \sigma_{ik} \, n_k \, x_j \, dS - \int_D \sigma_{ik,k} \, x_j \, dD \quad , \qquad (4)$$

The first integral on the right hand side is over the boundary of the body $|D|$, where $\sigma_{ik} \cdot n_k = 0$, (eqn. 2b), and the second integral, in the volume D, is zero as: $\sigma_{ik,k} = 0$, (eqn. 2a).

For a two-phase material (3) can be written as:

$$\int_D \sigma_{ij} \, dD = \int_{D-\Omega} \sigma_{ij} \, dD + \int_\Omega \sigma_{ij} \, dD = 0 \qquad (5)$$

where Ω is the volume of the second phase. Rewriting equation (5):

$$\frac{(D-\Omega)}{(D-\Omega)} \cdot \int_{D-\Omega} \sigma_{ij} \, dD + \frac{\Omega}{\Omega} \int_\Omega \sigma_{ij} \, dD = 0 \qquad (6\text{-a})$$

or:

$$(D-\Omega) \langle \sigma_{ij} \rangle_m + \Omega \langle \sigma_{ij} \rangle_p = 0 \qquad (6\text{-b})$$

multiplying both sides of 6-b by 1/D we obtain:[11]

$$(1-f) \langle \sigma_{ij} \rangle_m + f \langle \sigma_{ij} \rangle_p = 0 \qquad (6\text{-c})$$

where f is the volume fraction of the second phase and $\langle \sigma_{ij} \rangle_m$, $\langle \sigma_{ij} \rangle_p$ are the average stresses in the matrix and precipitates respectively.

Thus when average stresses obey Eqn. 6c, the true stress distribution at a point obeys Eqns. 2a, 2b.

It is clear, from the above arguments, that for two-phase materials, residual stress values obtained by x-rays could be substituted in Eqn. 6c to check if 2a, 2b are indeed obeyed. However, deviations may still occur since the stress carried by grain or phase boundaries is not taken into account.

This effect may cause an apparently non-zero value for eqn. 3 when residual stresses are measured by x-rays. Such stresses are called Pseudo-macrostresses and have been observed in single phase and multi-phase materials.[14]

B. Residual Stress Fields Resulting from Surface Deformation in Materials with "Hard" and "Soft" Regions

When a two phase material (where one of the phases is stronger) is subjected to surface deformation through processes such as shot-peening or grinding, the surface layers suffer plastic deformation ϵ_{ij}^{p}, with most of ϵ_{ij}^{p} occurring in the weaker phase. For example, in shot-peening, with the shot arriving normal to the surface, the (total) plastic strain ϵ_{ij}^{p} will be:

$$\left(\varepsilon_{ij}^{p}\right)_{\text{Shot-peen}} = \begin{pmatrix} \varepsilon_{11}^{p} & & 0 \\ & \varepsilon_{22}^{p} & \\ 0 & & \varepsilon_{33}^{p} \end{pmatrix} , \qquad (7)$$

where, from conservation of volume $\epsilon_{11}^{p} = \epsilon_{22}^{p} = -1/2\ \epsilon_{33}^{p}$.

These plastic stresses imply a length change in the surface layers which are elastically constrained by the bulk (where plastic deformation is limited, Fig. 4). Thus to a first approximation, we may represent the surface layers of the specimen as a slab of thickness h_1 and length L, where h_1 is the depth beyond which plastic deformation is nil. The length L represents the final length of the material after processing. The constraining effect of the bulk is taken into account by applying appropriate tractions $\underline{F}_i$ at the boundary of this equivalent slab (Fig. 5). These tractions will cause a "macro residual stress" field in the slab. (There will also be a micro residual stress field because of the plastic deformation differential between phases, which will be treated later.)

For shot-peening, the macrostresses in the surface layer will be:

$$\sigma_{11} = \sigma_{22} = E \cdot \ln \frac{L}{L_0 + \delta} , \qquad (8)$$

where L is the final length of the material, $L_o+\delta$ is the length the surface would attain if detached from the bulk and E is the Young's modulus of the composite material. Of course, in practice $L_o+\delta$ cannot be measured. However, we can find σ_{11}, σ_{22} by considering the equations of equilibrium over the total body (bulk and surface layers).

The stresses in the bulk after shot-peening will be:

$$(\sigma_{11})_{Bulk} = (\sigma_{22})_{Bulk} = E \cdot \ln \frac{L}{L_o} \quad , (9)$$

where L_o is the initial length of the body prior to shot-peening.

Since the net force on any plane cutting through the specimen must be zero, we have:

$$(\sigma_{ij})_{Bulk} \cdot (h_t - h_1) = (\sigma_{ij})_{Surface} \cdot h_1 \quad , (10)$$

here h_t is the total thickness of the material.

Thus, from Eqns. 9 and 10:

$$(\sigma_{ij})_{Surface} = E \cdot \frac{(h_t - h_1)}{h_1} \cdot \ln \frac{L}{L_o} \quad , (11)$$

for shot-peened materials.

The stresses given by eqn. (11) will be constant through the matrix and the precipitates in the surface layer. Thus their average value will be equal to their value at a point. If the surface layer is replaced by the equivalent slab* (Fig. 5a), the equations of equilibrium for this slab will be:

$$\sigma_{ij,j} = 0 \quad , \text{ in the slab} \quad , (2\text{-}a)$$

$$\sigma_{ij} \cdot n_j = F_i \quad , \text{ on the boundary} \quad , (12)$$

* This replacement is possible since h_1 is larger than the penetration depth of x-rays.

The forces $\underline{F}_i$, as stated earlier, represent the constraining effect of the bulk.

It is seen from Eqns. 12 and 4 that Eqn. 6c is not applicable to the macrostress given by Eqn. 11.

The microstress field occurs as a result of the differential plastic deformation between the matrix and the second phase. This field contains components that are rapidly varying with distance in the material and also components that are fairly constant with distance.[7,9,11,15] It has been shown that[10,11,15] when integrated over a volume containing a large number of second-phase particles, the rapidly fluctuating stresses average to zero, while the constant components yield finite values for each phase respectively. These values will satisfy Eqn. 6c, and are called "Pseudo-Macrostresses" (PMS) in the following discussion in keeping with the usual terminology of x-ray stress measurements.[14]

Since plastic deformation occurs only in the surface layers, the PMS field exist only in this surface layer. Thus, only the equivalent slab (Fig. 5) will be considered. To further simplify the solutions, we ignore the surface tractions $\underline{F}_i$ and calculate the PMS values for a slab with plastic strains ϵ_{ij}^{P} distributed uniformly through the volume. We also assume that these strains occur only in one phase. The total stress field will then be obtained from superposition of the PMS field and the macrostresses given by Eqn. 11. This procedure is approximate in that the stresses due to differences in elastic constants between the phases, caused by the macrostresses are neglected.* In the following treatment we follow procedures developed by Mura in reference

*When the elastic constants of the matrix and the precipitate are different, a given macrostress will cause different elastic strains in the phases. However, since displacements must be continuous across the boundary between phases an elastic stress field will arise.

(11).

Assume that this slab of total volume D has "N_0" precipitates that are elliptical in shape. The total volume of the precipitates is Ω. There are plastic strains ϵ_{ij}^{p} that are uniformly distributed in each precipitate. The equation of equilibrium for any stresses existing in the slab is given by Eqn. 6c.

The mean stress in the matrix $\langle \sigma_{ij} \rangle_m$ can be written as:

$$\langle \sigma_{ij} \rangle_m = C_{ijkl}\, e_{kl}^{o} \qquad , (13)$$

e_{kl}^{o} is the average elastic strain in the matrix. If a single precipitate is randomly inserted into the matrix (for large N_0 this will not change f). The stress in this new precipitate will be:(11)

$$\sigma_{ij} = \sigma_{ij}^{\infty} + \langle \sigma_{ij} \rangle_m \quad , \qquad (14)$$

Here σ_{ij}^{∞} is the stress calculated for a <u>single</u> precipitate, containing plastic strain ϵ_{ij}^{p}, present in an infinite matrix.

Since the precipitate can be inserted at any place in the matrix (we are assuming a random distribution of precipitates) σ_{ij}, given by equation (14) is the <u>average</u> stress in the precipitates.

Writing Hooke's law for the precipitate:

$$\sigma_{ij} = C_{ijkl}^{*} \left\{ e_{kl}^{o} + \epsilon_{kl} - \epsilon_{kl}^{p} \right\} \qquad , (15)$$

where e_{kl}^{o} is the average elastic strain in the matrix and ϵ_{kl} is the total strain in the precipitate containing plastic strains ϵ_{kl}^{p}. Thus the term in the brackets is the total elastic strain in the precipitate inserted into the

matrix.

Solution of equation (15) is possible through the equivalent inclusion method of Eshelby.[7,11]

Writing Hooke's law for the equivalent inclusion which has elastic constants C_{ijkl} (the same as the matrix):

$$\sigma_{ij} = C_{ijkl}\left(e^{o}_{kl} + \varepsilon_{kl} - \varepsilon^{p}_{kl} - \varepsilon^{*}_{kl}\right) , \quad (16)$$

Where ϵ^{*}_{kl} is the "eigenstrain"[11], that must be introduced into the equivalent inclusion such that:

$$C^{*}_{ijkl}\left(e^{o}_{kl} + \varepsilon_{kl} - \varepsilon^{p}_{kl}\right) = C_{ijkl}\left(e^{o}_{kl} + \varepsilon_{kl} - \varepsilon^{**}_{kl}\right) , \quad (17)$$

where:

$$\varepsilon^{**}_{kl} = \varepsilon^{p}_{kl} + \varepsilon^{*}_{kl} , \quad (18)$$

Assuming the matrix and the precipitate both to be isotropic*, Eqn. (17) becomes:

$$2\mu^{*}\left\{e^{o}_{ij} + \varepsilon_{ij} - \varepsilon^{p}_{ij}\right\} + \delta_{ij}\lambda^{*}\left\{e^{o}_{kk} + \varepsilon_{kk} - \varepsilon^{p}_{kk}\right\} =$$
$$2\mu\left\{e^{o}_{ij} + \varepsilon_{ij} - \varepsilon^{**}_{ij}\right\} + \delta_{ij}\lambda\left\{e^{o}_{kk} + \varepsilon_{kk} - \varepsilon^{**}_{kk}\right\} , \quad (19)$$

Here μ^{*}, λ^{*}, μ and λ are the Lame constants of the precipitate and matrix respectively.

Equation (18) can be expressed in a more useful form by defining the deviatoric strains:[11]

$$'e^{o}_{ij} = e^{o}_{ij} - \delta_{ij}e^{o}_{kk}/3$$

*For isotropic materials: $\sigma_{ij} = 2\mu\,\varepsilon_{ij} - \delta_{ij}\lambda\,\varepsilon_{kk}$, (17-b)

here $\varepsilon_{kk} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$, (17-c)

$$\grave{\varepsilon}_{ij} = \varepsilon_{ij} - \delta_{ij}\,\varepsilon_{kk}/3 \quad , \qquad (20\text{-}b)$$

$$\grave{\varepsilon}^{p}_{ij} = \varepsilon^{p}_{ij} - \delta_{ij}\,\varepsilon^{p}_{kk}/3 \quad , \qquad (20\text{-}c)$$

$$\grave{\varepsilon}^{**}_{ij} = \varepsilon^{**}_{ij} - \delta_{ij}\,\varepsilon^{**}_{kk}/3 \quad , \qquad (20\text{-}d)$$

Substituting Equation (20) in Eqn. (19) and re-arranging:[11]

$$2\mu^{*}\left\{\grave{e}^{o}_{ij} + \grave{\varepsilon}_{ij} - \grave{\varepsilon}^{p}_{ij}\right\} = 2\mu\left\{\grave{e}^{o}_{ij} + \grave{\varepsilon}_{ij} - \grave{\varepsilon}^{**}_{ij}\right\} \quad , (21\text{-}a)$$

$$K^{*}\left\{e^{o}_{kk} + \varepsilon_{kk} - \varepsilon^{p}_{kk}\right\} = K\left\{e^{o}_{kk} + \varepsilon_{kk} - \varepsilon^{**}_{kk}\right\} \quad , (21\text{-}b)$$

Where K^* and k are the bulk moduli of the precipitates and the matrix respectively: $(K = \lambda + 2\mu/3)$.

For shear strains $(i \neq j)$ Eqn. (21a) becomes:

$$2\mu^{*}\left\{e^{o}_{ij} + \varepsilon_{ij} - \varepsilon^{p}_{ij}\right\} = 2\mu\left\{e^{o}_{ij} + \varepsilon_{ij} - \varepsilon^{**}_{ij}\right\} \quad , (22)$$

since all $\delta_{ij} = 0$ for this case.

The unknowns ε_{ij}, ε^{**}_{ij} can be expressed in terms of each other by the property:[7,11]

$$\varepsilon_{ij} = S_{ijkl}\,\varepsilon^{**}_{kl} \quad , (23)$$

Where S_{ijkl} are the Eshelby tensors associated with the precipitate shape. Thus from Eqns. (22) and (23):

$$\varepsilon^{**}_{ij} = \left\{2(\mu-\mu^{*})\,e^{o}_{ij} + 2\mu^{*}\varepsilon^{p}_{ij}\right\} \Big/ \left\{4(\mu^{*}-\mu)\,S_{ijij} + 2\mu\right\} \quad , (24)$$

for $i \neq j$.

Another equation between the unknowns ϵ^{**}_{ij}, e^{o}_{ij}, is obtained by rewriting Eqn. (6c). This is done by substituting Eqn. (23) into Eqn. (16) and substituting the results and Eqn. (13) into Eqn. (6c), rewriting the final result for isotropic materials by using Eqn. (17b):

$$2\mu e^{o}_{ij} + \delta_{ij}\lambda e^{o}_{kk} + f\{2\mu(S_{ijkl}\varepsilon^{**}_{kl} - \varepsilon^{**}_{ij}) + \delta_{ij}\lambda(S_{11kl}\varepsilon^{**}_{kl} + S_{22kl}\varepsilon^{**}_{kl} + S_{33kl}\varepsilon^{**}_{kl} - \varepsilon^{**}_{kk})\} = 0 \quad ,(25\text{-a})$$

For shear strains $i \neq j$, we have:

$$2\mu e^{o}_{ij} + f\{2\mu(S_{ijkl}\varepsilon^{**}_{kl} - \varepsilon^{**}_{ij})\} = 0 \quad ,(25\text{-b})$$

from Eqns. (24) and (25b) we obtain:

$$e^{o}_{ij} = \varepsilon^{p}_{ij} \cdot \frac{2\mu^{*} \cdot f \cdot (1-2S_{ijij})}{4(\mu^{*}-\mu)S_{ijij} + 2\mu - f(1-2S_{ijij})(2\mu-\mu^{*})} \qquad (26)$$

for $i \neq j$.

The average shear stresses in the matrix are:

$$\langle\sigma_{ij}\rangle_{m} = 2\mu e^{o}_{ij} \quad , \quad (27)$$

Here e^{o}_{ij} is given by Eqn. 26. The average shear stresses in the precipitate are then found from Eqn. (6c).

Unfortunately the solution for normal stresses ($i=j$) is much harder: From Equations (19) and (23) we can obtain three equations in six unknowns e^{o}_{ij} and ϵ^{**}_{ij} (for $i=j$). Another three equations in e^{o}_{ij} and ϵ^{**}_{ij} are obtained from the

equilibrium Eqn. (25a). Then, e^{o}_{ij} and ϵ^{**}_{ij} can be obtained from the simultaneous solution of these six equations. Explicit solutions will not be given here; however, it can be seen by inspection that $\langle \sigma_{ij} \rangle_m$ for i=j will be a function of the plastic strain $\epsilon^{p}_{ij_{i=j}}$, the elastic constants of the matrix and precipitate and the Eshelby tensors S_{ijkl}.

DISCUSSION

A. Shot-Peening

If the surface of a two-phase material is shot-peened with the shot impinging in the direction of the surface normal $\underline{P}_3$ the plastic strains are given by equation (7). Thus from Eqn. 26, there will be no Pseudo-macro shear stresses. The surface layers will have Macro-stresses σ_{11}^{Macro}, σ_{22}^{Macro} given by Eqn. 11 and Pseudo. Macro stresses σ_{11}^{PM}, σ_{22}^{PM}, σ_{33}^{PM} from Eqn. 27.

The stresses in the surface; σ_{11}, σ_{22}, determined by x-rays will contain both components whereas any σ_{33} value will be a Pseudo-macro stress (Provided ϵ^{p}_{ij} is uniform with depth in the peened layer).

B. Grinding

Consider figure (6). For this case the grinding is along $\underline{P}_1$ in Fig. 2. The plastic strains at the surface layers will be:

$$\varepsilon^{p}_{ij} = \begin{pmatrix} \varepsilon^{p}_{11} & & \varepsilon^{p}_{13} \\ & \varepsilon^{p}_{22} & \\ & & \varepsilon^{p}_{33} \end{pmatrix} , \qquad (28)$$

The magnitudes of ϵ^{p}_{ij} depend on grinding parameters such as depth of cut, cooling, feed rate, etc. It can be seen from Fig. 6 that ϵ^{p}_{13} will change direction, when, for a given direction of wheel rotation, the feed direction is changed. From Eqn. 27, this will cause the pseudo-macro residual stress σ_{13} to change sign. Assuming that ϵ^{p}_{ij} in Eqn. (28) are uniformly distributed with

depth, the total stress tensor (determined by x-rays) at the surface layers will have macro components σ_{11}^{Macro}, σ_{22}^{Macro} and pseudo-macro components* $\sigma_{11}^{P.M}$, $\sigma_{22}^{P.M}$, $\sigma_{33}^{P.M}$ and $\sigma_{13}^{P.M}$.

If we examine the residual stresses produced by grinding in reference (3) for steels (2-phase materials) we see that there is a tri-axial stress state in the surface layers, in accordance with the predictions made above, the σ_{13} component of which changes sign with direction of feed, as predicted above. It was also found that the stress state calculated from the curvature of the bar, which occurred during grinding due to macro-residual stresses, does not agree with x-ray results, as the Pseudo-macro components, which affect the x-ray measurement, do not cause curvature.

It is also seen that straightening the samples, which will relax the macro-stress, do not affect σ_{13}. This is expected since straightening cannot produce $-\epsilon^{p}_{13}$, where ϵ^{p}_{13} was originally caused by the tangential force of the grinding wheel. For Armco Iron, ground with the same parameters as the steels, no σ_{13} is reported, and we predict none as this is a single-phase material. However, a σ_{33} value is reported. This value might be due to anisotropic plastic flow, where certain regions of the material do not flow plastically and act as "hard" regions. It might also be due to a residual error in the analysis since a low ψ- range (0-45°) was employed. It has been recently shown that when steep stress gradients are present in any component of the stress tensor, use of a high ψ-range (39-60) is more accurate.[19]

*The stresses that we call Pseudo-Macro stresses, following x-ray convention have also been called "Back Stresses" in studies of the Beuschinger effect.[18] This Back-stress has been observed in both multi-phase[14] and single-phase[20] materials.

CONCLUSIONS

1) In multi-phase materials, residual stresses must be measured in <u>all</u> phases if possible, if it is of interest to know if there are pseudo-macro stresses. If Eqn. (6c) is satisfied, the stress field consists of only pseudo-macro stresses.

2) Care must be taken in the application of differential equations of equilibrium to x-ray data. When only stresses in a single phase of a multi-phase material are reported, <u>no</u> conclusions can be derived from Eqns. 2a, 2b, with stresses obtained from x-ray analysis.

3) Shear stresses in the near-surface regions of ground two-phase materials do not necessarily violate any equilibrium equations and so, do not imply the presence of preferred orientation.

4) A Pseudo-Macro stress field exists in surface-deformed materials, even if a single phase, and this field may be measured. Its magnitude will depend on the relative strength of the phases (on load bearing regions), and on the amount of plastic deformation.

5) More results on the macro and pseudo-macro stresses in such materials would be of interest.

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CAPTIONS

Figure 1. Schematic of a diffractometer for Stress measurement.

a) Certain atomic planes satisfy Bragg's law and diffract x-rays at a 2θ value which depends on the spacing of the hkl planes. This spacing is affected by stresses.

b) After the specimen is tilted, diffraction occurs from other grains, but from the same set of planes. Since the normal stress component on these is different than in (a), the plane spacing will be different, as will the diffraction angle.

Figure 2. The definition of the specimen axes $\underline{P}_i$ which define the surface, and the measurement direction, $\underline{L}_{\phi\psi}$.

Figure 3. A piece-wise continuous stress distribution. The shaded regions represent the stress in precipitates and are invisible to x-rays if only a reflection from the matrix is examined.

Figure 4. Definition of surface layer (a) and schematic of (assumed) distribution of plastic deformation with depth in the sample after shot-peening (b).

Figure 5. The equivalent slab, with constraining forces representing the effect of bulk (a), and in relaxed state (b).

Figure 6. The forces acting in the wheel and in the surface layers during grinding. The feed is in the $\underline{P}_1$ direction of Fig. 2, it can be seen that if the feed direction is reversed, the tangential force F_t, causing ϵ_{13} will change sign.

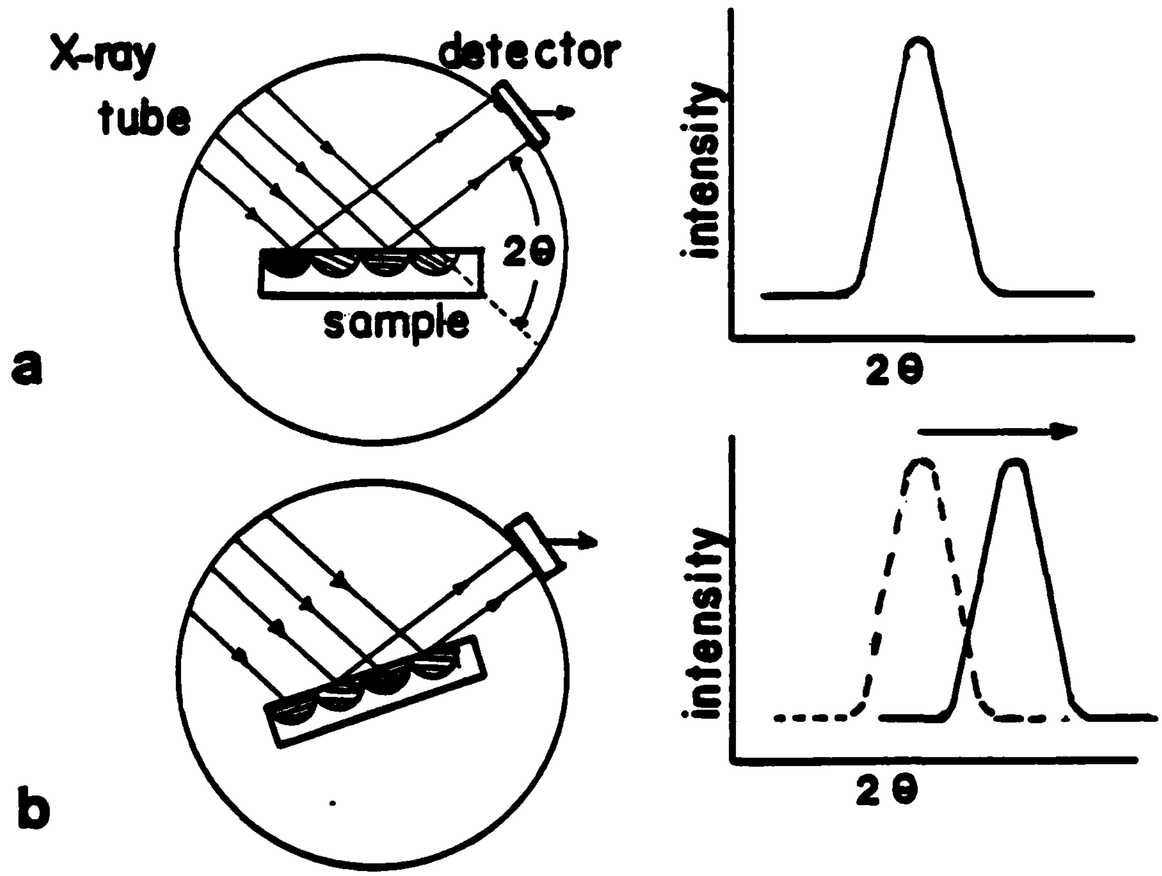

FIGURE 1

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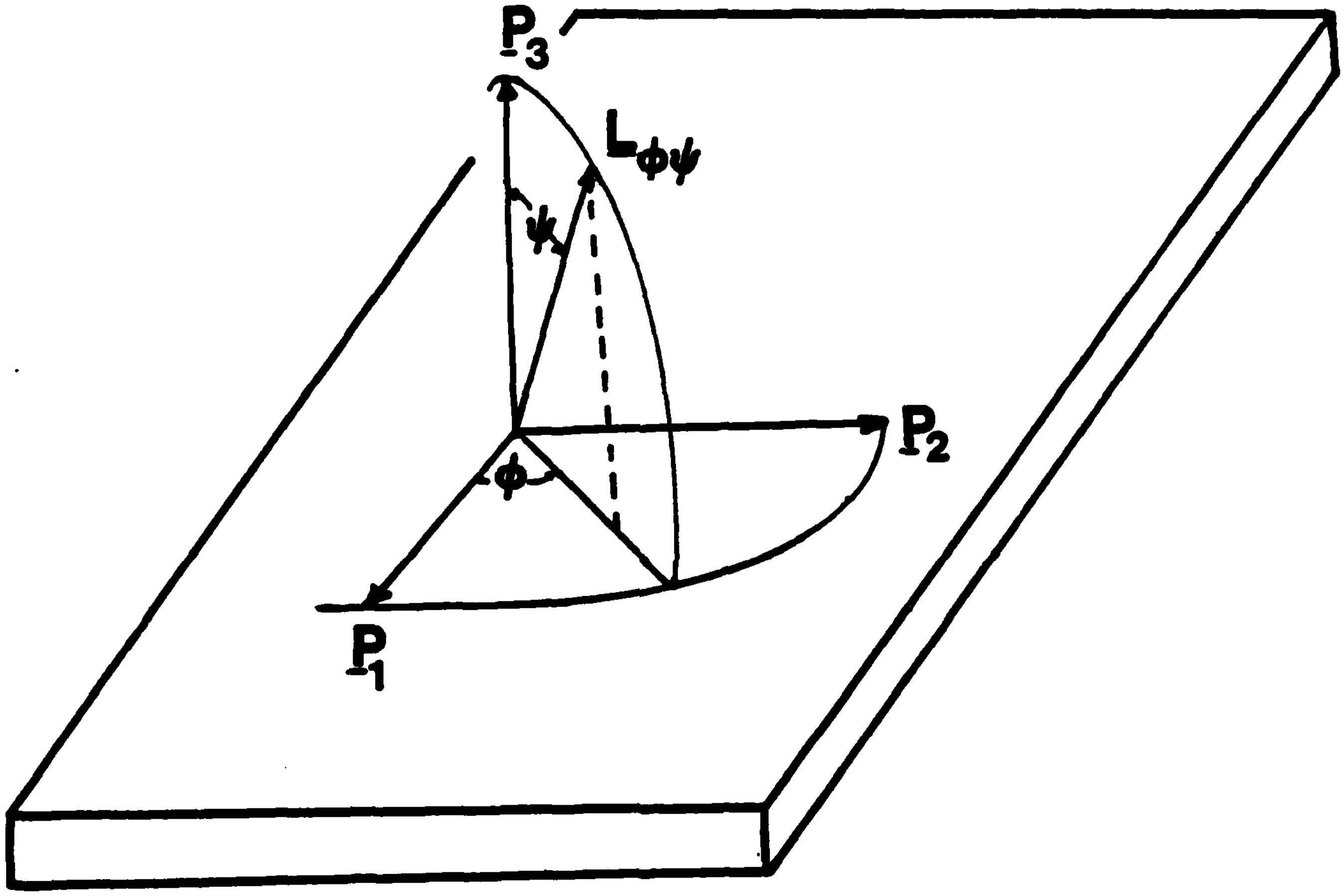

FIGURE 2

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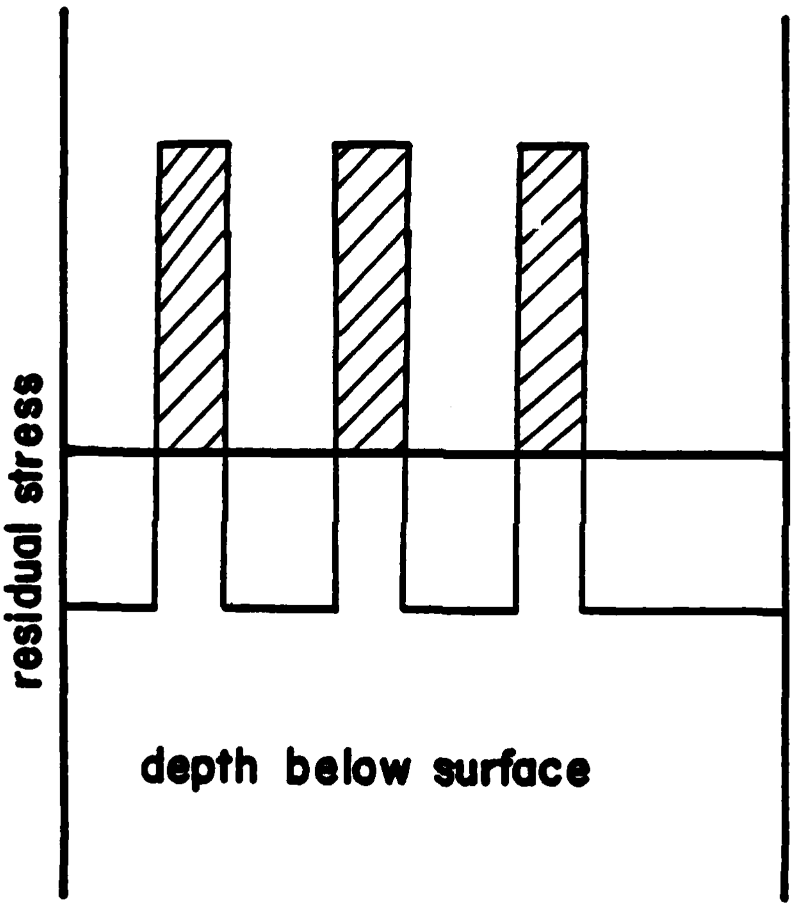

FIGURE 3

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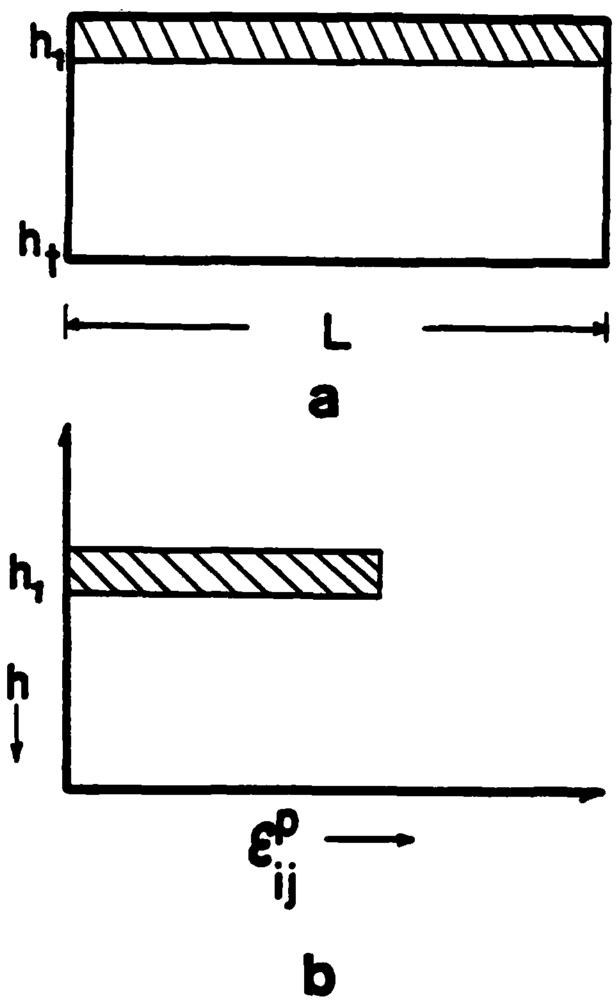

FIGURE 4

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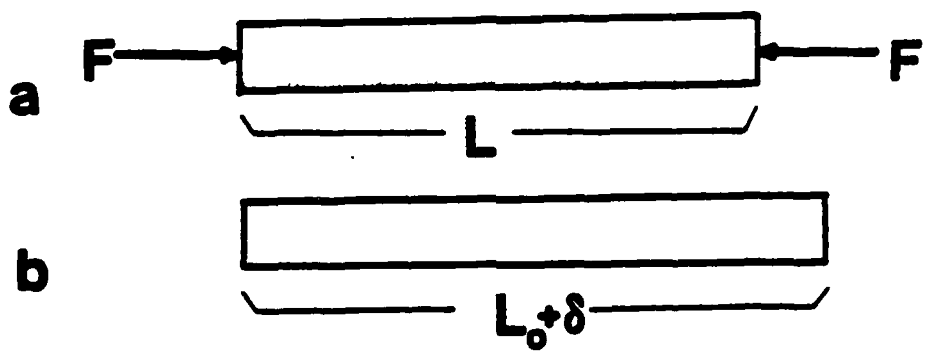

FIGURE 5

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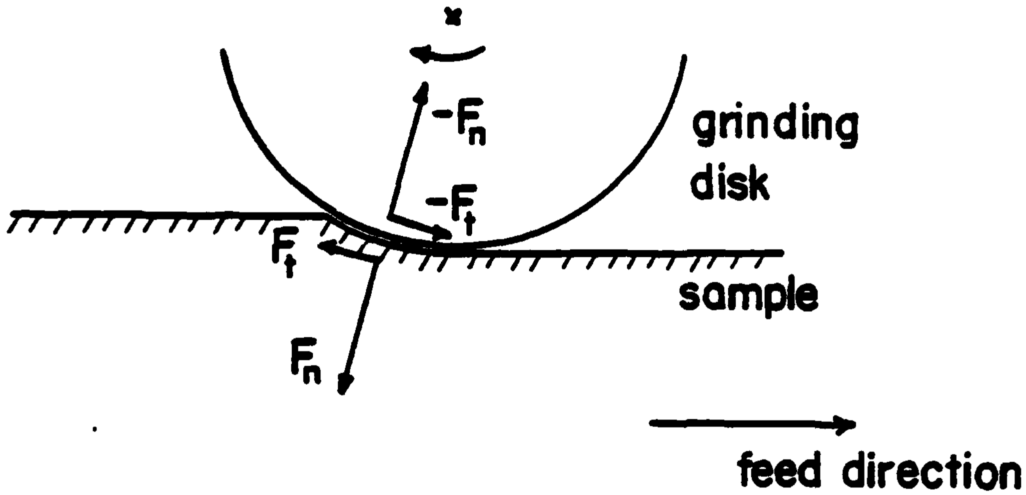

FIGURE 6

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13. ABSTRACT

True macro-stresses are measured with x-rays after material processing, only if there is a uniform plastic deformation in the sampled volume that is different than that in the rest of the material. In this stress-field the components in the direction of the surface normal are usually negligible over the depth penetrated by x-ray (for example, peening). However, pseudo-macrostresses, or backstresses, arise when this condition is violated, for example, if there are second phase particles and there is a gradient of plastic deformation from particle to matrix. This pseudo-macrostress field is three-dimensional and the stresses in the direction of the surface normal can be measured with x-rays. Equations are presented which allow estimates to be made of the magnitudes of the two kinds of stress fields.

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